Thermal Properties Calculations Using Finite-Difference Method

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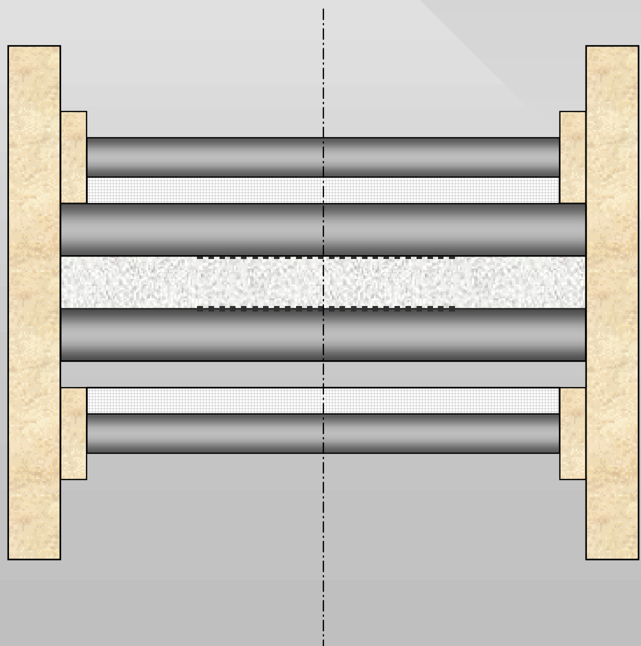
FOX Heat Flow Meter Instruments ASTM Standard C 518



- Two isothermal plates - hot and cold - at temperatures T_H and T_C
- Heat Flow Meters: Q_H , Q_C
- Flat sample of thickness DX
- Thermal conductivity λ
- Thermal diffusivity **a** - now can be determined as well

Regular Formula for the Heat Flow Meter Method

Steady-State - after reaching full thermal equilibrium - when Fourier number $Fo = at/DX^2 >> 1$

- $\lambda = DX/(T_H - T_C) *$
  $* [S_H(T_H) * Q_H + S_C(T_C) * Q_C]/2$
- average of the two heat flow meters
- S_H and S_C are calibration factors

Thermal Conductivity Equation (Non-Steady Transient State)

$$\partial^2 T(x,t)/\partial x^2 = (1/a)\, \partial T(x,t)/\partial t$$

- a - thermal diffusivity [m^2/s]
- Boundary conditions (B.C.):
- For hot plate (x=0) $T(0,t)=T_H(t)$
- For cold plate (x=DX) $T(DX,t)=T_C(t)$
- Initial conditions (I.C.) (t=0, 0<x<DX): $T(x,0)$

One-dimensional temperature field evolution with time: T(x,t)

- Initially, sample has some initial temperature distribution T(x,0) (usually, room temperature).
- After placing flat sample between the instrument's plates, its inner temperature distribution T(x,t) starts to change according to the thermal conductivity equation, eventually reaching the final thermal equilibrium condition.
- This transient process contains information about both thermal conductivity and diffusivity.

Thermal Conductivity Equation using Finite-Difference Method:

- $[T(x+\delta x, t) - 2T(x,t) + T(x-\delta x, t)] / (\delta x)^2 \cong$ $\cong (1/a)$ [**<u>T(x, t+δt)</u>** – T(x,t)] / δ t
- <u>Next</u> moment temperatures **T(x, t+δt)** can be calculated using <u>previous</u> moment temperatures T(x+δx, t),T(x,t), and T(x-δx, t)
- <u>Boundary conditions:</u>
- T(0,t)=TH(t);   T(DX,t)=TC(t)
- $q_H(t)=\lambda[T(0,t)-T(\delta x,t)]/\delta x = S_H*Q_H(t)$
- $q_C(t)=\lambda[T(DX,t)-T(DX- \delta x,t)]/\delta x=S_C*Q_C(t)$

Two pairs of the heat flow arrays:

- 1) Experimental: $Q_H(t)$ and $Q_C(t)$
- 2) Calculated: $Q_{Hcalc}(t)$ and $Q_{Ccalc}(t)$ which are calculated using two input parameters - values of thermal conductivity λ and thermal diffusivity **a**. Their correct values can be found using Least-Squares Method (weighted):
- $F(\lambda,a)=\Sigma\{[Q_H-Q_{Hcalc}]^2+[Q_C-Q_{Ccalc}]^2\}m^2$ sum of squares of differences $\Rightarrow$ minimum

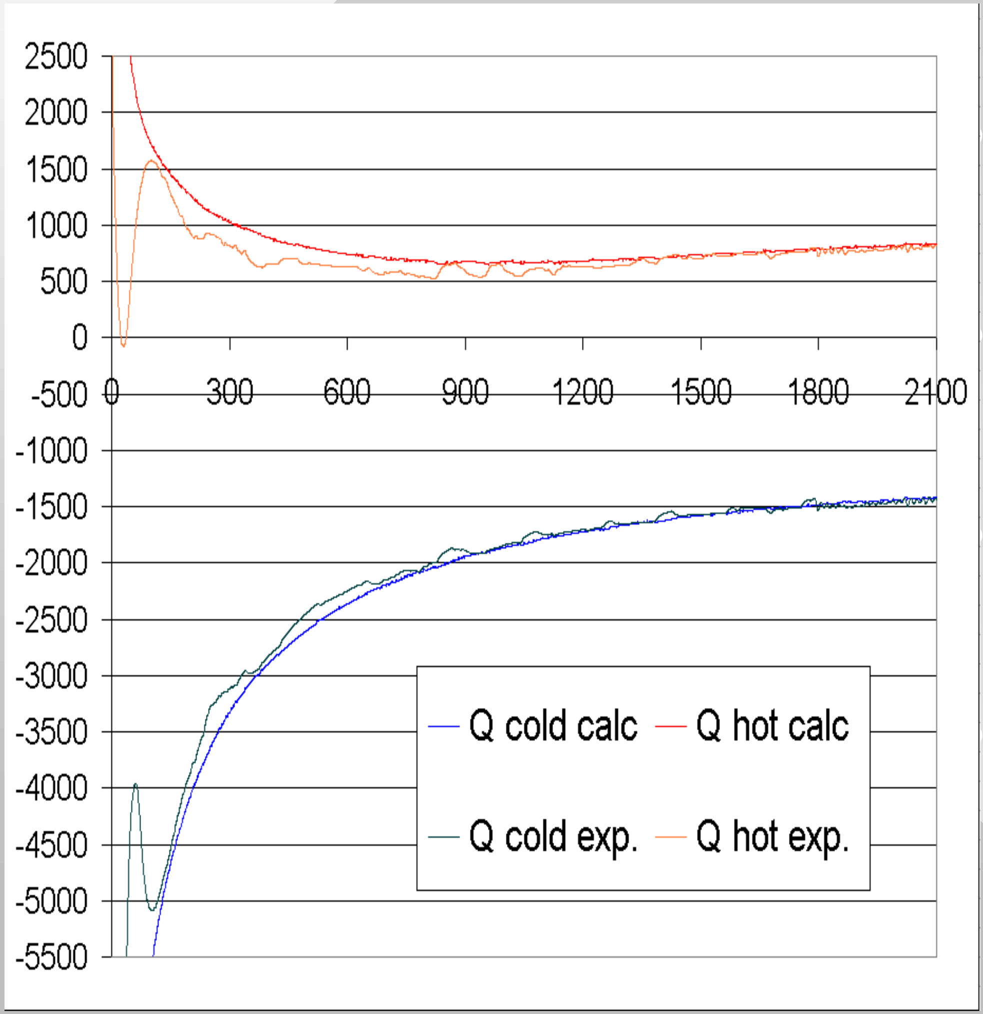
$F(\lambda,a) \Rightarrow$ minimum

- Any analytical function has minimum if all (two in our case) its partial derivatives are equal to zero:
- $f_\lambda = \partial F(\lambda,a)/\partial\lambda = 0$
- $f_a = \partial F(\lambda,a)/\partial a = 0$
- Newton's method is most useful and convenient to find the <u>best pair of λ and **a**</u> values using their initial guess values and iteration calculation procedure.

Newton's method:

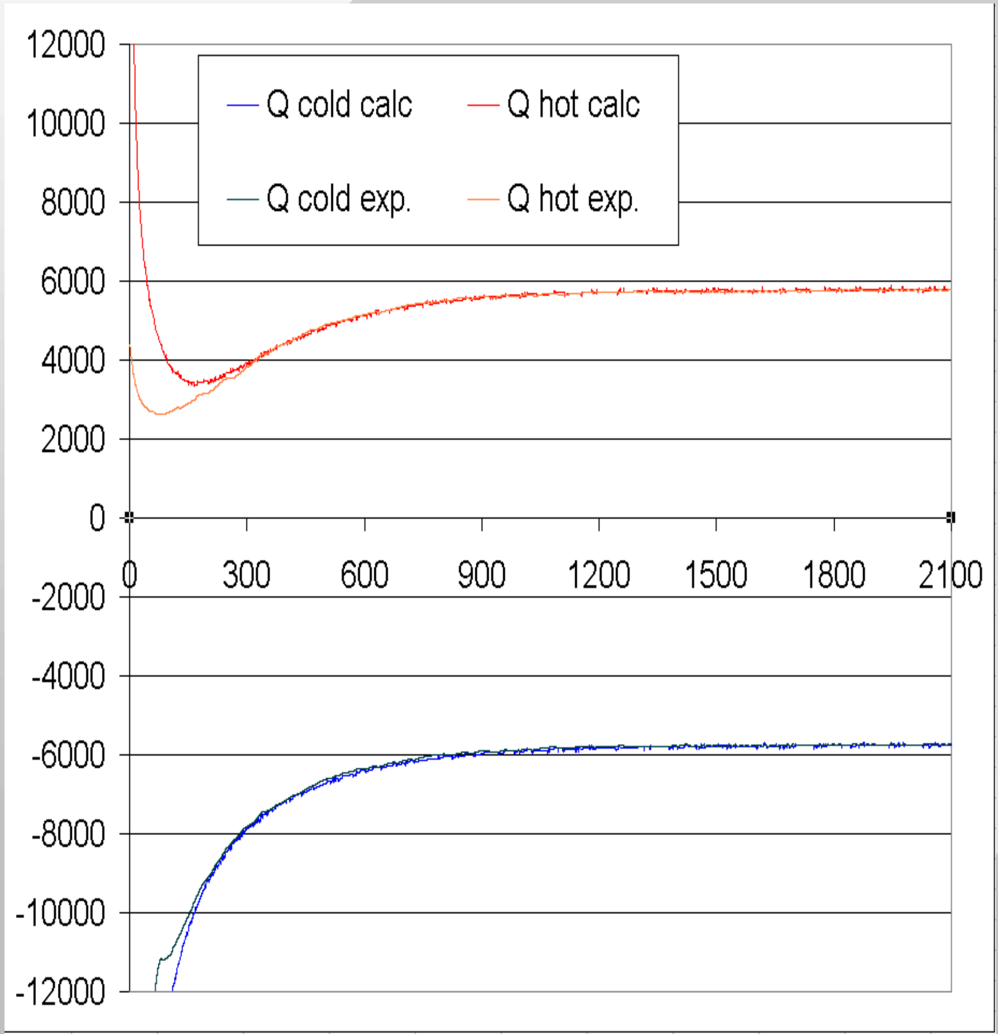
- $f_\lambda + (\partial f_\lambda / \partial \lambda)[\lambda^{(j+1)} - \lambda^{(j)}] + (\partial f_\lambda / \partial a)[a^{(j+1)} - a^{(j)}] = 0$
- $f_a + (\partial f_a / \partial \lambda)[\lambda^{(j+1)} - \lambda^{(j)}] + (\partial f_a / \partial a)[a^{(j+1)} - a^{(j)}] = 0$
- System of two linear equations with two unknowns: $\lambda^{(j+1)}$ and $a^{(j+1)}$:
- 1) $(\partial f_\lambda / \partial \lambda)\lambda^{(j+1)} + (\partial f_\lambda / \partial a)a^{(j+1)} =$
  $= (\partial f_\lambda / \partial \lambda)\lambda^{(j)} + (\partial f_\lambda / \partial a)a^{(j)} - f_\lambda;$
- 2) $(\partial f_a / \partial \lambda)\lambda^{(j+1)} + (\partial f_a / \partial a)a^{(j+1)} =$
  $= (\partial f_a / \partial \lambda)\lambda^{(j)} + (\partial f_a / \partial a)a^{(j)} - f_a;$

Heat Flow Meters Signals - calculated and experimental (in microvolts) versus time (in seconds) - 1"-thick "good" vacuum panel



- Thermal conductivity $\lambda = 5.79$ mW/(m*K)
- Thermal diffusivity $a = 4.3 \cdot 10^{-8}\ m^2/s$
- Volumetric specific heat $C_p\rho = 135\ kJ/(m^3K)$
- Thermal effusivity $\varepsilon = 27.9\ W\ sec^{1/2}/(m^2K)$

Heat Flow Meters Signals - calculated and experimental (in microvolts) versus time (in seconds) - <u>1”-thick “bad” vacuum panel</u>



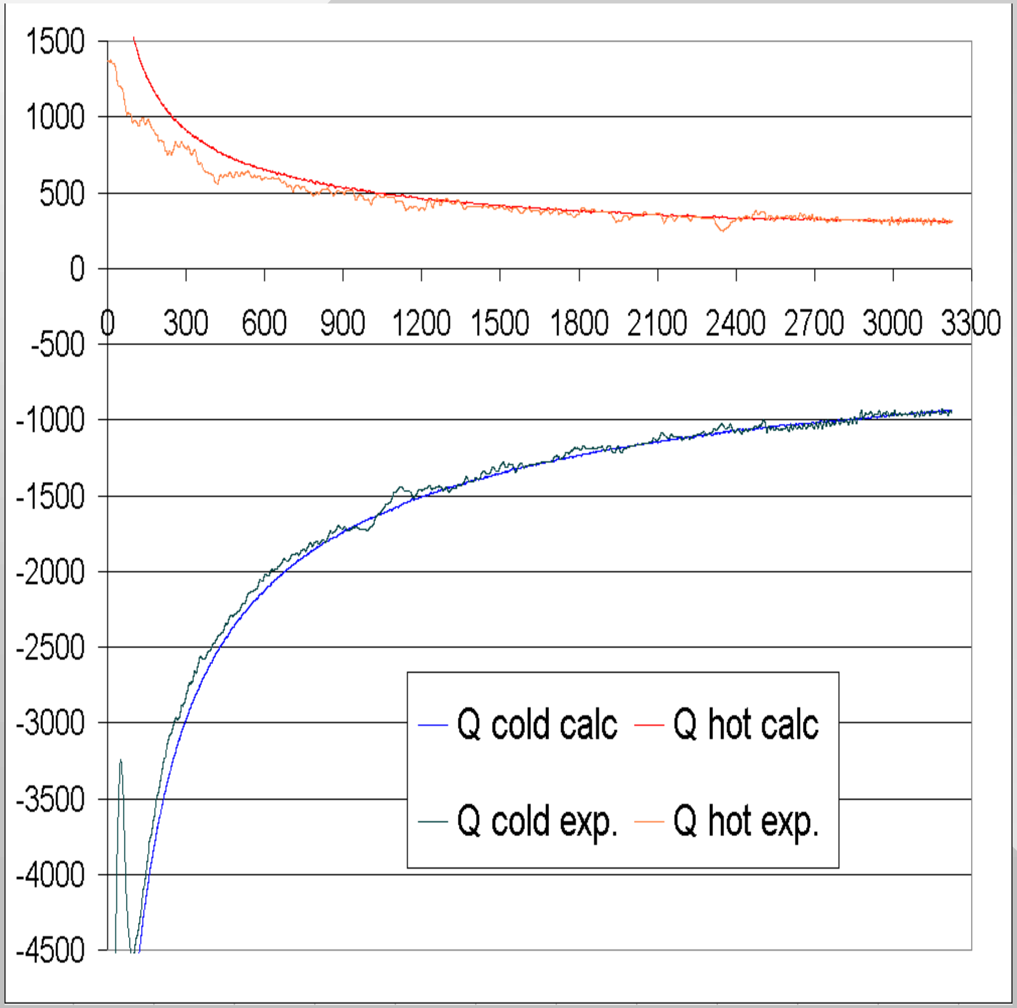
Thermal conductivity λ= 33.3 mW/(m*K)

Thermal diffusivity $a=2.9*10^{-7}$ m^2/s

Volumetric specific heat $C_p\rho$= 115 kJ/(m^3K)

Thermal effusivity ε= 61.8 W $sec^{1/2}$/(m^2K)

Heat Flow Meters Signals - calculated and experimental (in microvolts) versus time (in seconds) - <u>2”-thick “good” vacuum panel</u>



Thermal conductivity λ=5.23 mW/(m*K)

Thermal diffusivity a=$4.3*10^{-8}$ m^2/s

Volumetric specific heat $C_p\rho$=122 kJ/(m^3K)

Thermal effusivity ε=25.2 W $sec^{1/2}$/(m^2K)

New mathematical algorithm of calculations was developed

- Two thermal properties - thermal conductivity λ and thermal diffusivity **a** can be calculated long before reaching full thermal equilibrium.
- Also two more thermal properties can be calculated - volumetric specific heat $\mathbf{C_p\rho=\lambda/a}$ and thermal effusivity $\boldsymbol{\varepsilon=\lambda/\sqrt{a}}$

Prospective

- The new algorithm will be used in the LaserComp's "WinTherm" software for FOX family of the Heat Flow Meter instruments.
- Tests duration can be made many times shorter using the new algorithm. This will be especially efficient in case of vacuum super-insulation panels and thick samples of thermal insulation materials.