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Taking test

instruments

into the

21st century

• Dynamic Heat Flow Meter for Quick Thermal Properties Measurements Using Finite-Difference Method

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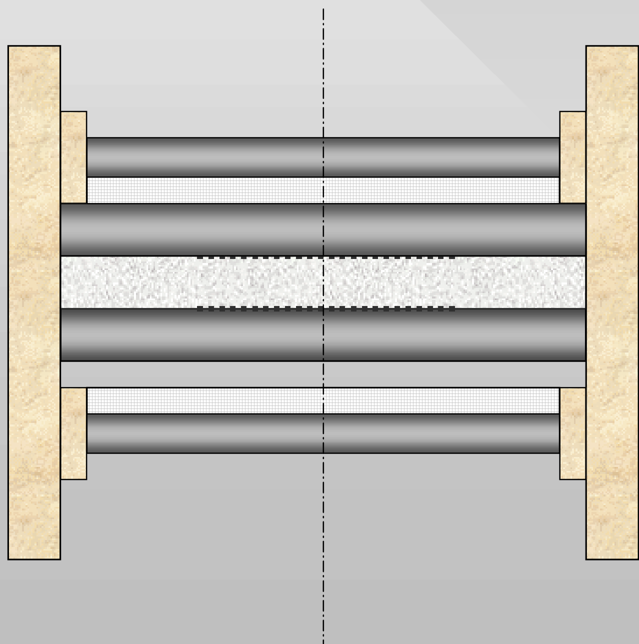
(not in the Symposium's program because of the late submission)

Heat Flow Meter Instruments ASTM Standard C 518 (FOX Instruments - LaserComp)



- Two isothermal plates -
  hot and cold -
  at temperatures T_H and T_C
- Heat Flow Meters: Q_H , Q_C
- Flat sample of thickness DX
- Result -thermal conductivity λ after reaching final thermal equilibrium

Regular Heat Flow Meter Method

Traditional Steady-State method – thermal conductivity λ is determined after reaching full thermal equilibrium - when Fourier number $Fo = at/DX^2 >> 1$

Typical value of thermal diffusivity a for insulation materials is ~$3*10^{-7}$ m^2/s , so 10 cm (or 4”)-thick sample needs at least 6-8 hours.

- $\lambda = DX/(T_H - T_C)*$
  $*[S_H(T_H)*Q_H + S_C(T_C)*Q_C]/2$
- average of the two heat flow meters
- S_H and S_C are calibration factors

One-dimensional temperature field evolution with time: T(x,t)

- Initially, the flat-parallel sample has some known one-dimensional temperature distribution $T_0(x,0)$.
- After placing the sample between the HFM instrument's isothermal plates, the sample's temperature distribution T(x,t) starts to change according to the thermal conductivity equation, eventually reaching the final thermal equilibrium condition.
- **This dynamic process contains information about 4 thermal properties (2 independent).**

Dynamic HFM – Effusivity Probe

- Sample initially has known temperature distribution $T_0(x)$ (e.g. room temperature)
- Heat Flow Meter is bonded to an isothermal metal plate at temperature $T_1 \sim 15^0$-20^0C lower (or higher) than T_0
- After the Heat Flow Meter is placed on the sample to be tested the sample's inner temperature T(x, t) starts to change, and signal of the Heat Flow Meter is recorded (x – coordinate, t –time)
- Thermal effusivity $\boldsymbol{\varepsilon = \lambda / a^{1/2}}$ of the sample is determined – this can be **the quickest vacuum super-insulation panels QC method**

Thermal Conductivity Equation

$$\partial^2 T(x,t)/\partial x^2 = (1/a)\, \partial T(x,t)/\partial t$$

a - thermal diffusivity [m^2/s]

- Initial conditions (I.C.) ($0<x<\infty$, $t=0$):
  $T(x, t=0)=T_0$
  (sample has known initial temperature)

- Boundary conditions (B.C.):

$T(x=0, t)=T_1$ (metal plate's temperature) for all t

$T(x\rightarrow\infty, t)=T_0$ ("semi-infinite" sample)

Analytical Solution of the Thermal Problem for Semi-Infinite Body:

- $[T(x,t)-T_1]/(T_0-T_1) = \mathrm{erf}\,[x/(4at)^{1/2}]$
  where **erf** is Gaussian error function

- Heat flux q/A [W/m^2] at the surface (x=0):

  $$q/A(x=0, t)= \varepsilon\,(T_1 - T_0)/\,(\pi t)^{1/2}$$

  where $\varepsilon=\lambda/a^{1/2}$ is thermal effusivity
  (Schneider, P.J.:"Conduction Heat Transfer," Addison-Wesley Publishing Company Inc., Reading, Massachusetts, 1955)

- **Thermal effusivity ε is proportional to the slope of the heat flux vs. $1/\sqrt{t}$ graph**

Analytical Formula Limits

- Analytical formula is not valid at initial moments of time when the Heat Flow Meter's Fourier number
  $Fo' = a't/x'^{2}$ is not >>1
  (x'–HFM's thickness, a'–it's thermal diffusivity)
- Analytical formula is not valid after the thermal disturbance reaches the back surface of the sample - when the sample's Fourier number
  $Fo = at/x^{2}$ is not <<1
  (x–sample's thickness, a–it's thermal diffusivity)

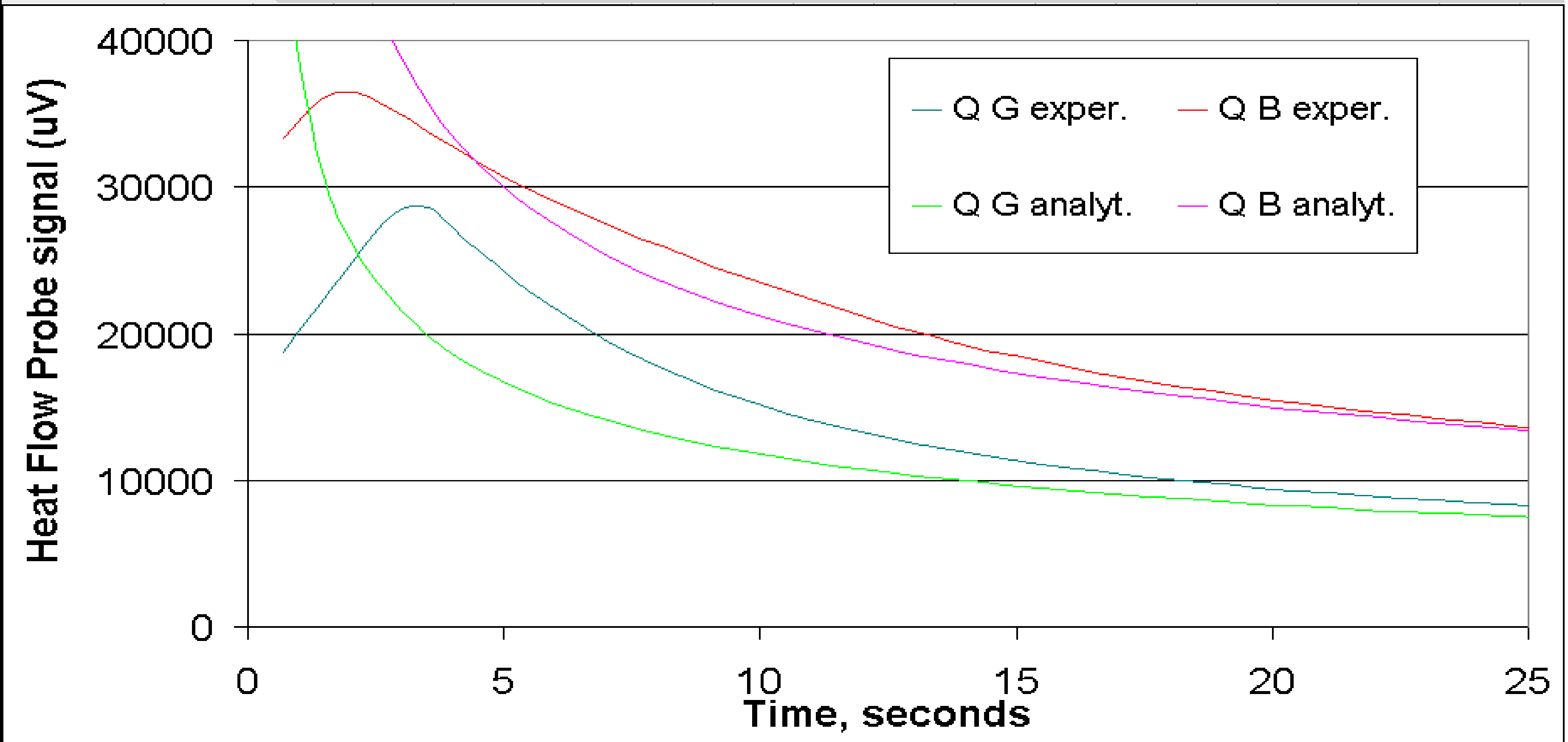
•I.e. the Analytical Formula is valid only during some time “window” - between t_{min} and t_{max} . This “window of opportunity” is ideal for quick Vacuum Super-Insulation Panels (VIP) Q.C.:

- For our 1 mm-thick Heat Flow Meter made of FR-4 resin ($a=1.037*10^{-7}$ m^2/s)
  t_{min} >~8 seconds

- For 1”-thick “bad” VIP ($a \sim 2.9*10^{-7}$ m^2/s)
  t_{max} < ~42 seconds
- For 1”-thick “good” VIP ($a \sim 4.3*10^{-8}$ m^2/s)
  t_{max} < ~282 seconds (~5 minutes)

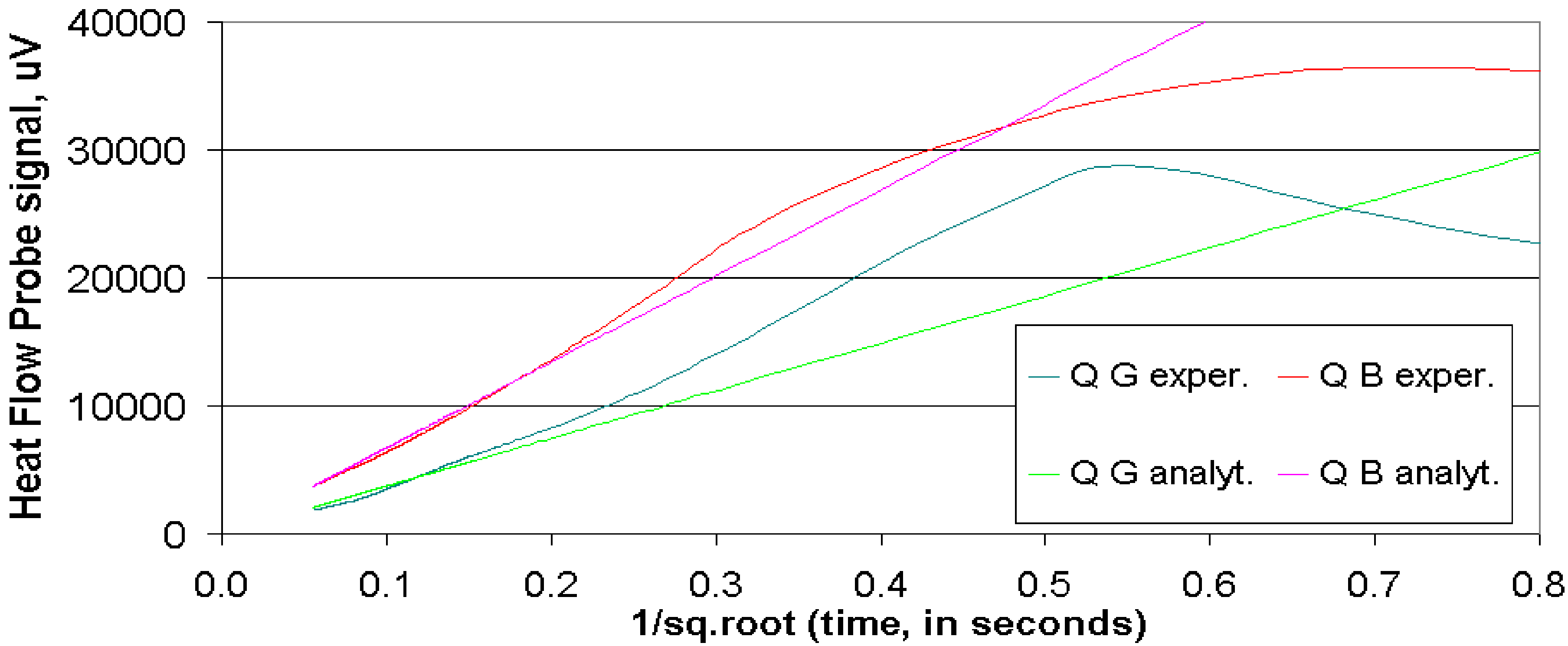
Experimental and Calculated using Analytical formula Dynamic HFM signals (μV) vs. time (seconds). ($T_0=21^0C; T_1=6^0C$)

- "Good" VIP (G): ε_G ~25 W $sec^{1/2}/(m^2K)$; λ_G =0.0056 W/mK;
- "Bad" VIP (B): ε_B ~45 W $sec^{1/2}/(m^2K)$; λ_B =0.0320 W/mK;



Experimental and Calculated using Analytical formula Dynamic HFM signals (μV) vs. 1/sq.root(time). ($T_0=21^0C$; $T_1=6^0C$)

- "Good" VIP (G): ε_G ~25 W $sec^{1/2}/(m^2K)$; λ_G =0.0056 W/mK;
- "Bad" VIP (B): ε_B ~45 W $sec^{1/2}/(m^2K)$; λ_B =0.0320 W/mK;



4 sec. match to 0.5; 8 sec.- to 0.354; 25 sec.- to 0.2; 100 sec.- to 0.1

Thermal effusivity vs. time - determined using the Dynamic HFM signal and the Analytical Formula. ($T_0=21^0C$; $T_1=6^0C$)

- "Good" VIP (G): ε_G ~25 W $sec^{1/2}/(m^2K)$; λ_G =0.0056 W/mK;
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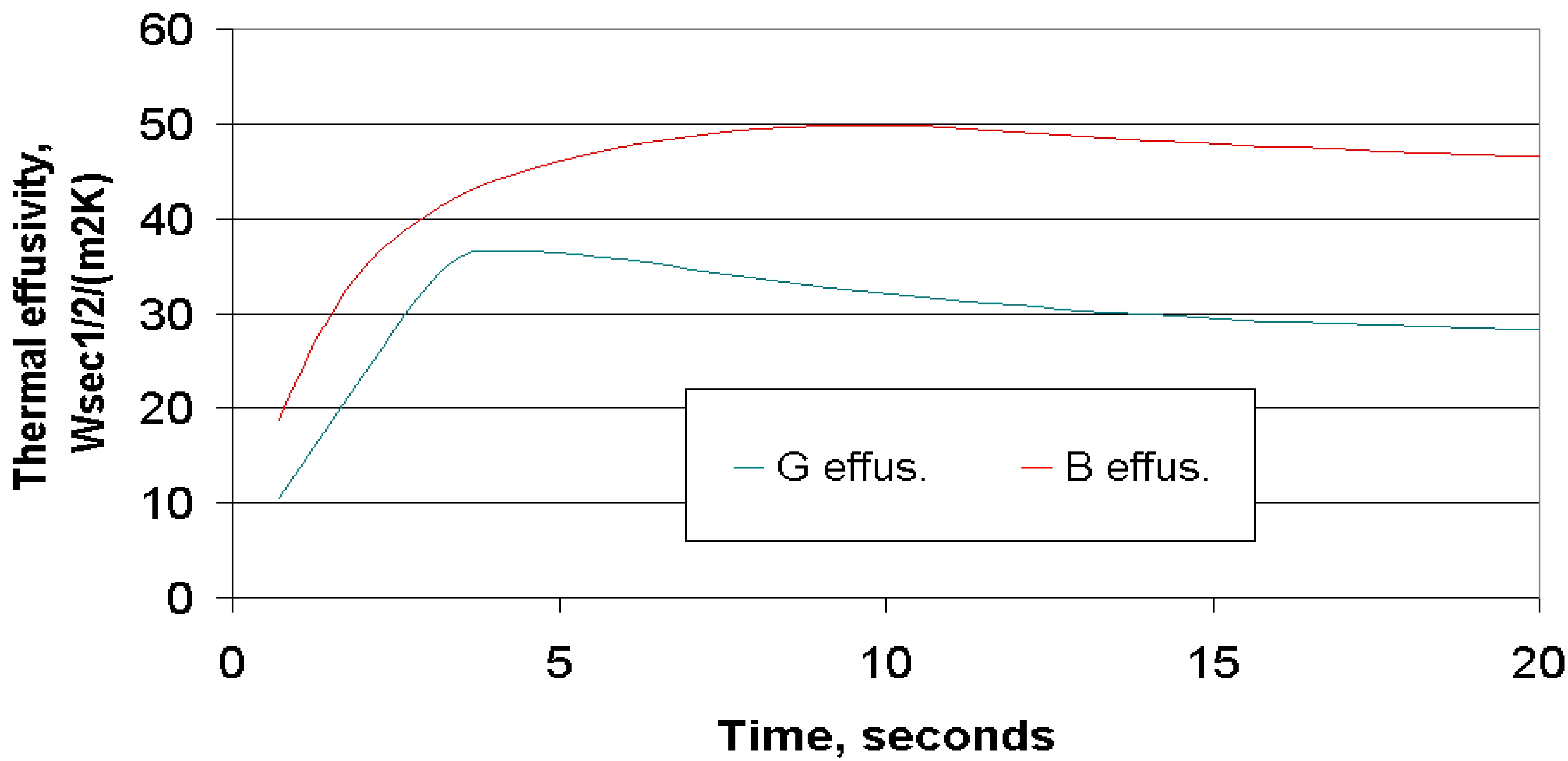


- We see very satisfactory agreement between the experimental and the calculated (using the Analytical Formula) Dynamic HFM's signals. Deviation between them is at the first moments of time t<~8 seconds when Fourier number

  $Fo' = a't/x'$ is not >>1

  ($a' = 1.037 \times 10^{-7}$ m^2/s; $x' = 1$ mm)

- To find a <u>more informative numerical solution</u> of the thermal problem we will use <u>**the Finite Difference Method.**</u>

Thermal Conductivity Equation using Finite-Difference Method:

- $[T(x+\delta x, t) - 2T(x,t) + T(x-\delta x, t)] / (\delta x)^2 \cong$
  $\cong (1/a)$ [**<u>$T(x, t+\delta t)$</u>** $- T(x,t)] / \delta t$

- <u>Next</u> moment temperature **$T(x, t+\delta t)$** for every point x can be calculated using <u>previous</u> moment temperatures $T(x+\delta x, t)$,$T(x,t)$, and $T(x-\delta x, t)$ at the point x, and 2 adjacent points $x+\delta x$ and $x-\delta x$

- Temperatures and heat flux at all co-ordinates and at all moments of time $T(x_i , t_i)$ can be calculated starting from the initial condition at $t=0$

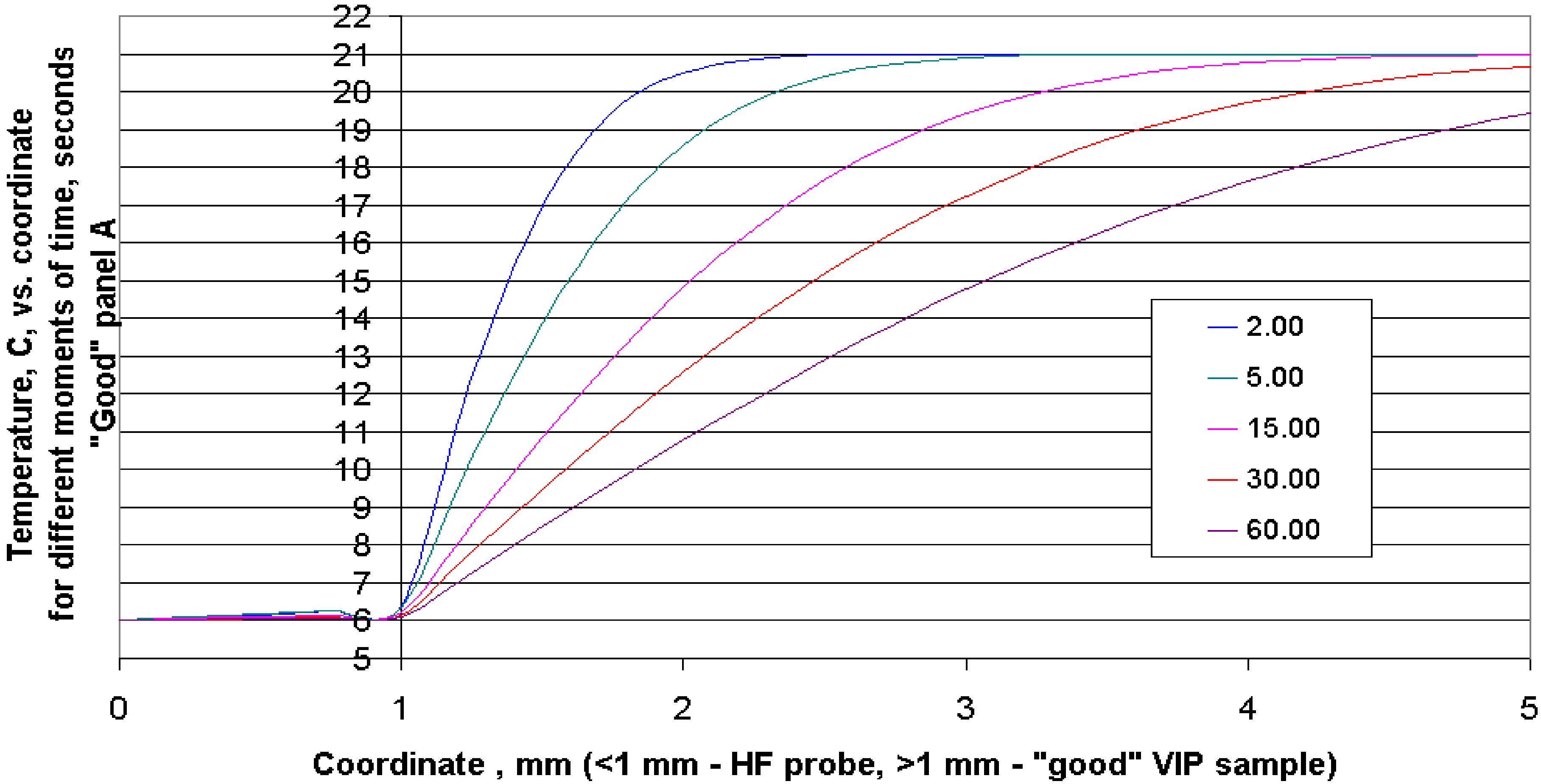
Initial and Boundary Conditions:

- Initial condition:
- $T(0,x)=T_0(x)$ for $t=0$ and $0<x<DX$
- Boundary conditions:
- $T(0,t)=TH(t)$ hot plate temperature
- $T(DX,t)=TC(t)$ cold plate temperature
- $q_H(t)=\lambda[T(0,t)-T(\delta x,t)]/\delta x = S_H * Q_H(t)$
- $q_C(t)=\lambda[T(DX,t)-T(DX-\delta x,t)]/\delta x = S_C * Q_C(t)$

Temperature Evolution Calculated using the Finite Difference Method (part of the Microsoft Excel table)

T HFM	40.00		**T init.**	25	**ThCond.**	**0.035**	**ThEff.**	**45**	**ThDiff.**	**6.05E-07**	**At 40C:**		
HFM thick	**0.0010**	**No of nodes in l**		**1**	**ThCond.H**	**0.293**	**ThEff.HFM**	**9.10E+02**	**ThDiff.HFM**	**1.04E-07**	**S cal U =**	**0.005322**	**S cal L =**
Time		**Step**	**0.001**	**Time inte**	**0.703**								
	0	1	2	3	4	5	6	7	8	9	10	11	12
0.00	**40.00**	**40**	**25**	**25**	**25**	**25**	**25**	**25**	**25**	**25**	**25**	**25**	**25**
0.70	**40.00**	**38.21**	31.38	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1.41	**40.00**	**39.18**	31.57	27.71	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
2.11	**40.00**	**39.09**	33.17	28.20	26.15	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
2.81	**40.00**	**39.29**	33.57	29.44	26.53	25.49	25.00	25.00	25.00	25.00	25.00	25.00	25.00
3.52	**40.00**	**39.32**	34.25	29.96	27.33	25.73	25.21	25.00	25.00	25.00	25.00	25.00	25.00
4.22	**40.00**	**39.39**	34.58	30.66	27.77	26.19	25.34	25.09	25.00	25.00	25.00	25.00	25.00
4.92	**40.00**	**39.43**	34.96	31.10	28.33	26.50	25.59	25.16	25.04	25.00	25.00	25.00	25.00
5.62	**40.00**	**39.47**	35.22	31.56	28.73	26.89	25.79	25.29	25.07	25.02	25.00	25.00	25.00
6.33	**40.00**	**39.49**	35.47	31.91	29.15	27.21	26.05	25.41	25.14	25.03	25.01	25.00	25.00
7.03	**40.00**	**39.52**	35.67	32.25	29.50	27.54	26.27	25.57	25.21	25.07	25.02	25.00	25.00
7.73	**40.00**	**39.54**	35.85	32.53	29.84	27.83	26.51	25.71	25.30	25.11	25.03	25.01	25.00
8.44	**40.00**	**39.56**	36.01	32.80	30.13	28.12	26.73	25.88	25.39	25.16	25.05	25.02	25.00
9.14	**40.00**	**39.58**	36.15	33.03	30.41	28.39	26.96	26.04	25.50	25.21	25.08	25.03	25.01
9.84	**40.00**	**39.59**	36.28	33.24	30.66	28.64	27.17	26.20	25.61	25.28	25.11	25.04	25.01
10.55	**40.00**	**39.60**	36.40	33.44	30.90	28.88	27.38	26.36	25.72	25.35	25.15	25.06	25.02
11.25	**40.00**	**39.62**	36.50	33.62	31.12	29.10	27.58	26.52	25.83	25.42	25.20	25.08	25.03
11.95	**40.00**	**39.63**	36.60	33.78	31.32	29.31	27.78	26.68	25.95	25.50	25.24	25.11	25.04
12.65	**40.00**	**39.64**	36.69	33.94	31.51	29.52	27.97	26.84	26.07	25.58	25.30	25.14	25.06
13.36	**40.00**	**39.65**	36.77	34.08	31.69	29.71	28.15	26.99	26.19	25.67	25.35	25.17	25.08
14.06	**40.00**	**39.66**	36.85	34.21	31.86	29.89	28.32	27.14	26.31	25.76	25.41	25.21	25.10
14.76	**40.00**	**39.66**	36.92	34.33	32.02	30.06	28.49	27.29	26.43	25.84	25.47	25.25	25.12
15.47	**40.00**	**39.67**	36.99	34.45	32.17	30.22	28.65	27.43	26.55	25.93	25.53	25.29	25.15
16.17	**40.00**	**39.68**	37.05	34.56	32.31	30.38	28.80	27.57	26.66	26.02	25.60	25.33	25.18
16.87	**40.00**	**39.69**	37.11	34.66	32.45	30.53	28.95	27.71	26.78	26.12	25.67	25.38	25.20
17.58	**40.00**	**39.69**	37.16	34.76	32.57	30.67	29.09	27.84	26.89	26.21	25.74	25.43	25.24
18.28	**40.00**	**39.70**	37.22	34.85	32.70	30.81	29.23	27.97	27.00	26.30	25.80	25.48	25.27
18.98	**40.00**	**39.70**	37.27	34.94	32.81	30.94	29.37	28.10	27.11	26.39	25.87	25.53	25.31

Temperature inside the "good" VIP vs. co-ordinate (mm) calculated for different moments of time (sec) using the Fin.Diff.Method



Two pairs of the heat flow arrays:

- 1) Experimental: $Q_H(t)$ and $Q_C(t)$
- 2) Calculated: $Q_{Hcalc}(t)$ and $Q_{Ccalc}(t)$ which are calculated using two input parameters - values of thermal conductivity λ and thermal effusivity ε. Their correct values can be found using Least-Squares Method (weighted):
- Residue function

  $F(\lambda,\varepsilon)=\Sigma\{[Q_H-Q_{Hcalc}]^2+[Q_C-Q_{Ccalc}]^2\}m^2$

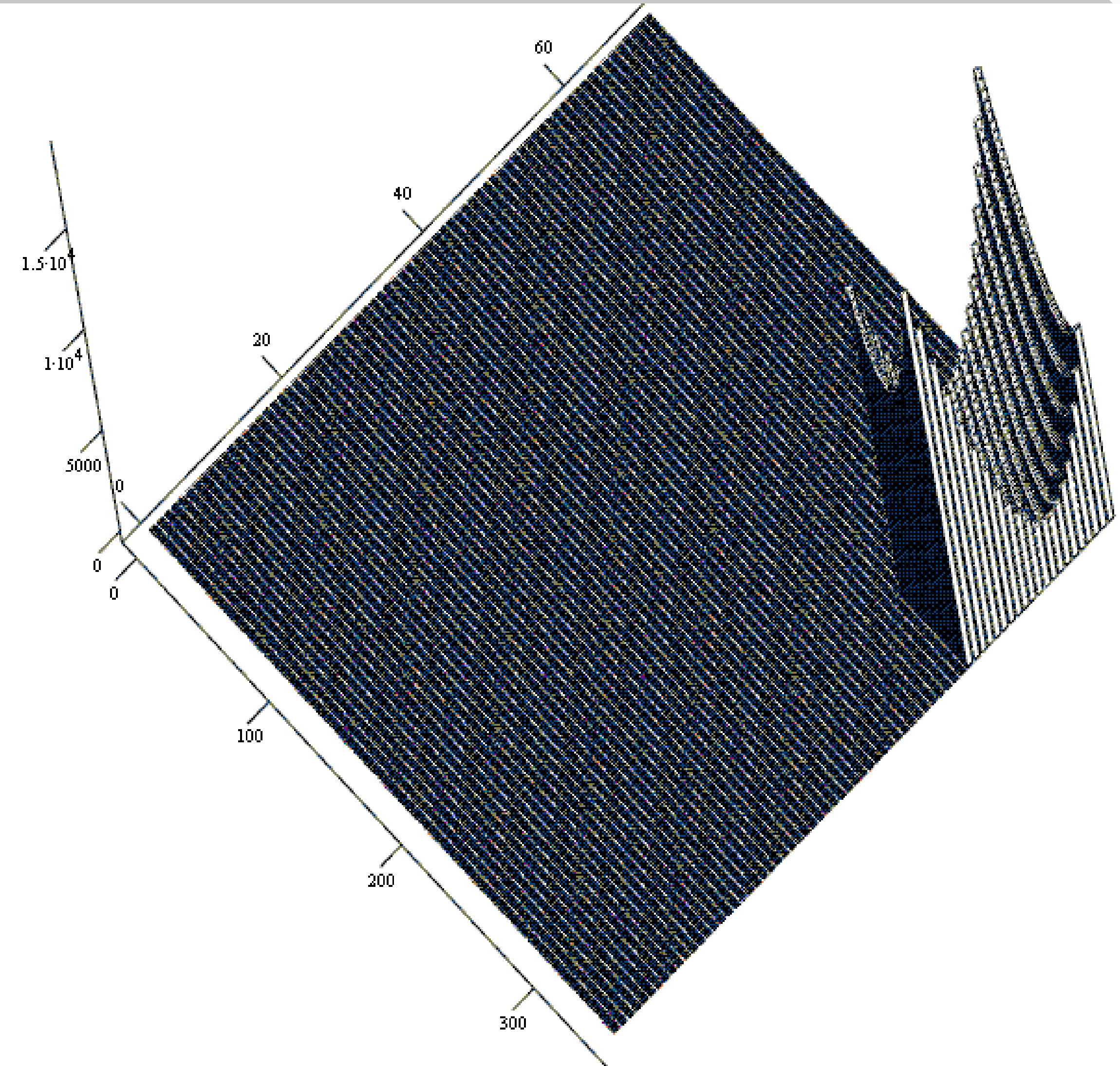
  sum of squares of differences $\Rightarrow$

  $\Rightarrow$ minimum

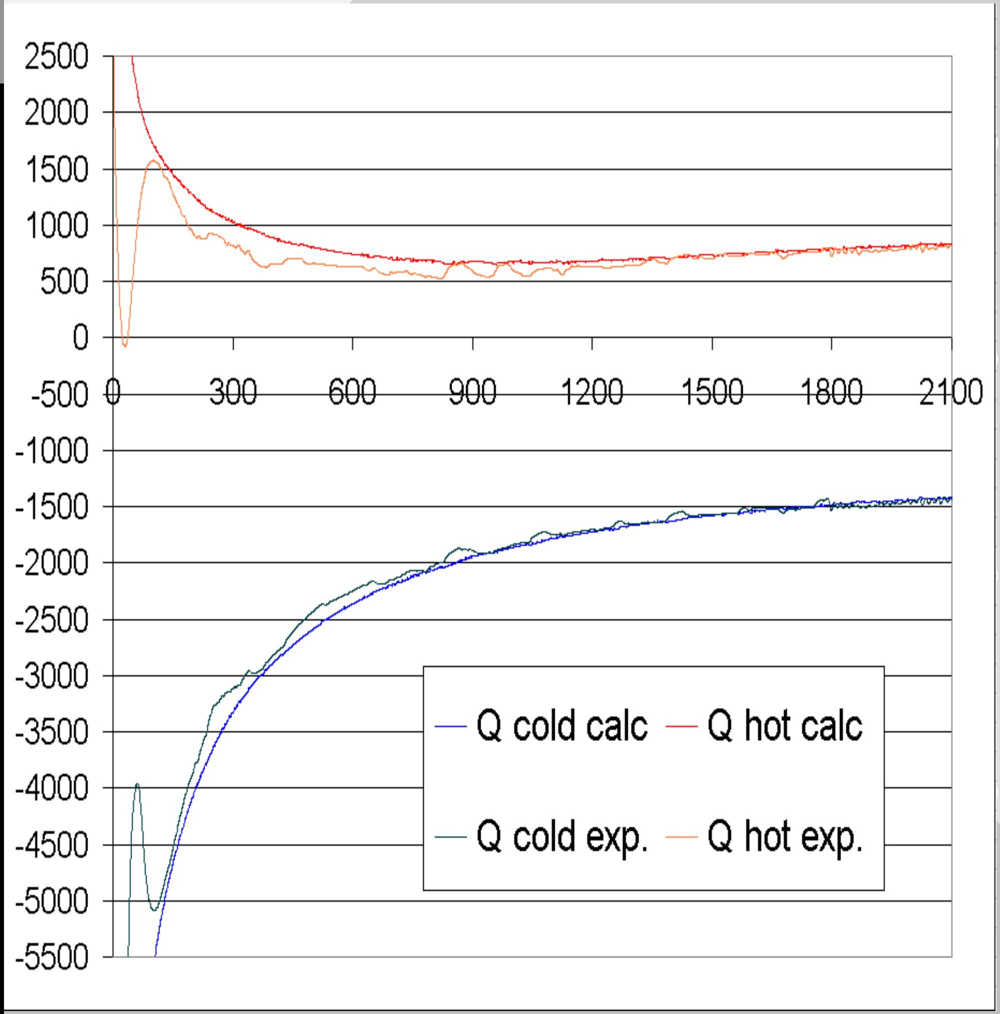
$F(\lambda, \varepsilon) \Rightarrow$ minimum

- Any analytical function has minimum
- It is possible to find the best pair of λ and ε values using their initial guess values and iteration computation procedure.
- Why λ and ε? Because the process of thermal relaxation in the sample
  - at the beginning is determined only by ε,
  - at the end – only by λ,
  - in the middle – by both ε and λ.

- Three-dimensional graph of the residue function of the two thermal properties – thermal conductivity and thermal effusivity (only the minimum zone is shown).
- The minimum corresponds to the best found pair of the thermal properties
λ=0.0333 mW/mK
ε=61.8 $Wsec^{1/2}/(m^2K)$



Heat Flow Meters Signals - calculated and experimental (in microvolts) versus time (in seconds) - <u>1"-thick "good" vacuum panel</u>



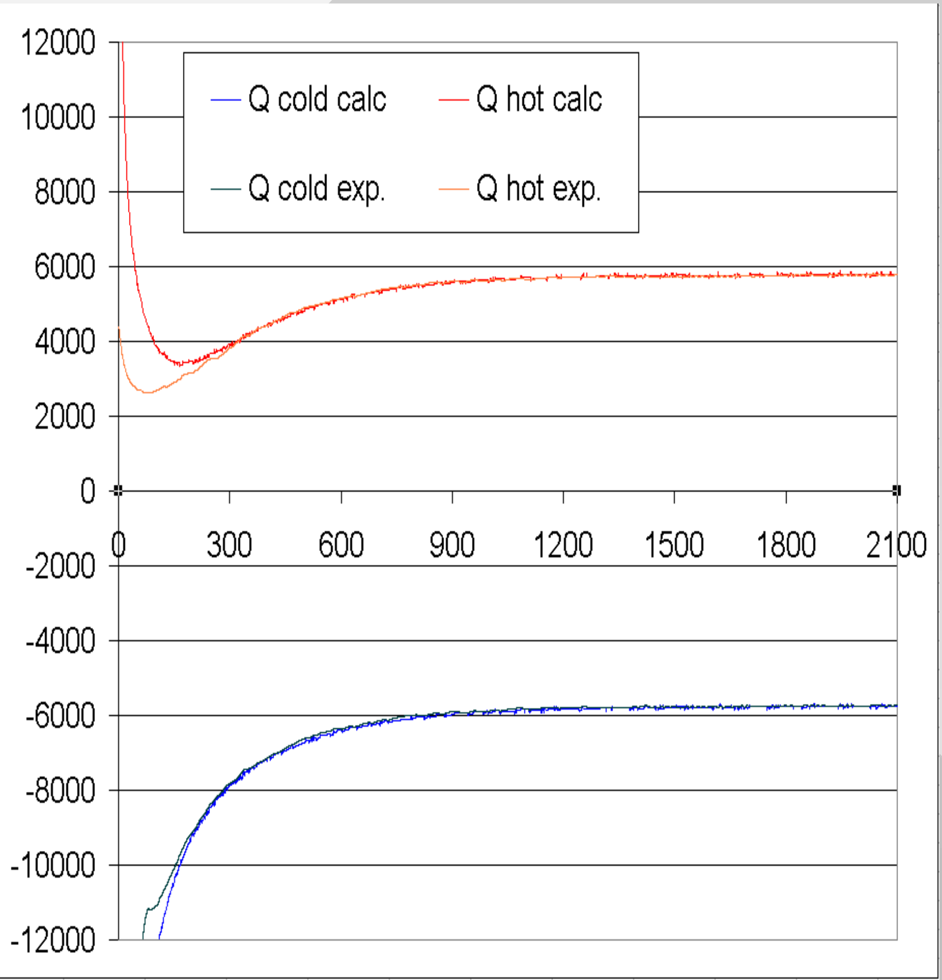
Thermal conductivity
λ= 5.79 mW/(m*K)

Thermal diffusivity
a=4.3*10^{-8} m^2/s

Volumetric specific heat
$C_p\rho$=135 kJ/(m^{3}K)

Thermal effusivity
ε=27.9 W sec$^{1/2}$/(m^{2}K)

Heat Flow Meters Signals - calculated and experimental (in microvolts) versus time (in seconds) - 1”-thick “bad” vacuum panel



- Thermal conductivity λ= 33.3 mW/(m*K)
- Thermal diffusivity a=$2.9*10^{-7}$ m^2/s
- Volumetric specific heat $C_p\rho$= 115 kJ/(m^3K)
- Thermal effusivity ε= 61.8 W $sec^{1/2}$/(m^2K)

Analytical Formula vs. Finite Difference Method

- Analytical Formula can be used only for thermal effusivity calculations when the Dynamic HFM Probe is "thermally thin" (i.e. when $Fo`=a`t/x`^2 >> 1$) and sample is "thermally thick" (sample's $Fo = at/x^2 << 1$)
- Finite Difference Method extracts information about all 4 thermal properties of the sample (2 of them are independent) after the sample becomes thermally not semi-infinite for the HFMs. Least-Squares method should be used to minimize the residual function.

New method for quick VIP QC was developed and checked

- Thermal effusivity $\varepsilon = \lambda / \sqrt{a}$ of the VIP (or other insulation material) can be determined <u>a few seconds</u> after touching the Dynamic HFM to the surface of the sample.

- <u>No waiting time or pause</u> is necessary between the tests because the HFM does not need to return to cooler temperature. This enables thousands of VIPs to be tested during one working day using only one Dynamic HFM device.

- Thermal resistance (if sample's thickness is known) and all other thermal properties can be determined <u>within a few minutes</u> as well (provided the sample is placed on isothermal plate).

New mathematical algorithm of calculations was developed

- Two thermal properties - thermal conductivity λ and thermal effusivity $\boldsymbol{\varepsilon=\lambda/\sqrt{a}}$ can be calculated long before reaching full thermal equilibrium.
- Also two more thermal properties can be calculated - thermal diffusivity $\boldsymbol{a=\lambda^2/\varepsilon^2}$ and volumetric specific heat $\boldsymbol{C_p\rho= \varepsilon^2/\lambda}$

Prospective

- The new Dynamic Heat Flow Meter method will be used in a new LaserComp's device for quick VIP QC.
- The new Dynamic Heat Flow Meter method will be used in the existing LaserComp's FOX Instruments to make the tests duration dramatically shorter. This will be especially efficient in case of thick samples of thermal insulation materials and vacuum super-insulation panels.
- All other thermal properties in addition to thermal conductivity will be possible to measure now simultaneously – thermal diffusivity, thermal effusivity and volumetric specific heat.