Conductive and Radiative Heat Transfer in Insulators

Akhan Tleoubaev, Ph.D.
LaserComp, Inc., December 1998

Heat transfer for most thermal insulation materials occurs via both conduction and radiation. Thermal radiation has long been recognized as an important mechanism of heat transfer in glass-fiber and other low-density insulation materials. Convection, previously thought to be a significant component of the thermal performance of these materials, was shown to be negligible.

Low-density thermal insulation materials, which permit significant heat transfer by radiation, show some increase in thermal conductivity (or decrease thermal resistance value per unit thickness) as thickness is increased. This so-called "thickness effect" in glass fiber building materials for a change from 1.5 to 6.0 inches (3.8 to 15.2 cm) was found to be in the range of 1.5% to 3% [1]. Some authors contend that radiation effects are responsible for 30 to 50 percent of the total heat transfer in low-density fibrous insulations. The "thickness effect" usually appears only at the low-density (less than ~30 kg/m^3 or 1.9 lb/ft^3) materials. The higher-density foam materials (e.g. Expanded Polystyrene – EPS, which has density about 55.4 kg/m^3 or 3.45 lb/ft^3) generally are too dense to show the effect at moderate temperatures [2].

Equipment used by vast majority of laboratories, including National Institute of Standards and Technology – NIST (former National Bureau of Standards - NBS), are able to measure only 1-inch (2.54 cm) samples. Relatively few laboratories have capabilities to test at greater thicknesses. The few apparatuses capable of testing at greater thicknesses were limited to test at 4 inches at most. NIST provided reference standards whose thermal conductivity was determined for 1-inch samples only [2].

Fig.1 illustrates the "thickness effect". Apparent thermal conductivity of polyurethane samples of different thickness measured by FOX200 instruments is plotted versus thickness for four values of temperature. Density of the material is about 28 kg/m^3.

When two infinite parallel plates are considered, all the radiation leaving one plate reaches the other one and the radiative heat flow per unit area (heat flux) between the plates is equal to [3]:

$$q_{rad} = \sigma(T^4{}_1 - T^4{}_2) / [2/\varepsilon - 1] \cong 4\varepsilon^2 \sigma T^3 \Delta T$$

ENCO SAMPLES TESTS VS. THICKNESS (THICKNESS EFFECT)

5/18/99

	5 mm	10 mm	20 mm	30mm	50 mm
	EN5_230	**EN10_230**	**EN20_230**	**EN30_216**	**EN50_235**
	5	**10**	**20**	**30**	**50**
12.5C	0.03179	0.0323	0.03272	0.03325	0.03388
22.5C	0.03305	0.0336	0.03408	0.03443	0.03514
32.5C	0.03436	0.03497	0.03551	0.03581	0.03651
42.5C	0.03569	0.03639	0.03701	0.03732	0.03801

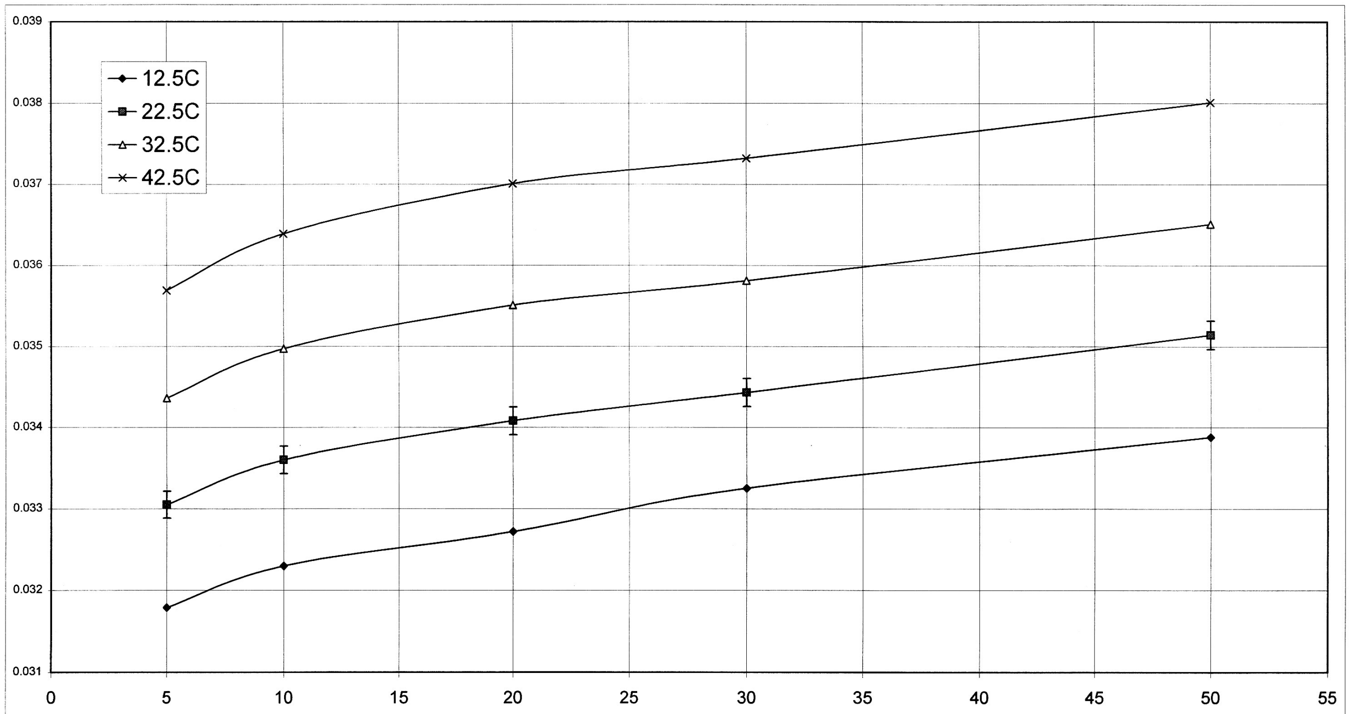

Fig.1

where ε is emissivity of the surfaces ($\varepsilon \cong 0.9$ for blackened surfaces, supposed to be same for both plates), σ is Stefan-Boltzmann constant (5.67×10^{-8} W m^{-2} K^{-4}), T is absolute mean temperature (K), ΔT is temperature difference (K), ($\Delta T << T$). Note that the radiative heat flux depends on the temperature difference ΔT and does not depend on the distance Δx between the plates.

Conductive heat flux between the parallel plates without radiation is equal to:

$$q_{cond} = \lambda_{cond} (\Delta T/\Delta x)$$

where λ_{cond} is conductive ("true") thermal conductivity of the material placed between the plates, ΔT is temperature difference, Δx is distance between the plates.

Conductive heat flux becomes much larger than radiative one at small values of the Δx (at constant ΔT). Therefore, measuring apparent thermal conductivity at different Δx and extrapolating it to zero thickness $\Delta x \rightarrow 0$ one can exclude the radiative deposit and get true conductive value of thermal conductivity of the material. This is the most reliable, simple and widely used way to determine the true value of thermal conductivity.

Under steady-state conditions, the apparent thermal conductivity of a slab of material of thickness Δx is normally computed as

$$\lambda = q \ \Delta x / \Delta T = (q_{cond} + q_{rad}) \Delta x / \Delta T$$

where ΔT is the temperature difference corresponding to the measured heat flux.

Heat transfer through thermal insulation materials can be considered as having following components:

- true thermal conduction through the continuous and discontinuous phases;
- thermal radiation;
- thermal convection within the pores or cells of the material (for materials with small cells or pores heat transfer due to convection usually can be neglected).

Radiative heat transfer through insulation can involve both:

1) scattering of thermal radiation from the solid phase of the insulation, and

2) absorption of radiant energy within the insulation, with an attendant change in temperature and re-radiation of energy.

For simultaneous conduction and radiation in a medium there are two limiting cases for which simple solutions can be obtained:

At one extreme, it is assumed that all of the heat transmitted through a slab of material flows in accordance with Fourier's law, i.e. the heat flux is proportional to an effective thermal conductivity times the spatial temperature gradient:

$$q_{cond} = \lambda_{eff}(dT/dx)$$

This situation can occur either:
1) if all of the heat is transferred by true conduction, or
2) if some of the heat is radiated, absorbed, and re-radiated with a mean free path that is shorter than specimen thickness (so-called "optically thick" specimen).

In this case both heat fluxes are coupled and an effective thermal conductivity can be used in computing the total heat flux.

At the other extreme, it is assumed that the conductive heat flux is in accordance with Fourier's law but that the radiative heat flux takes place with scattering but without absorption or radiation in the medium. In this latter case the heat flux by radiation is uncoupled from that by conduction so, the two heat fluxes can be computed separately and added [4].

Rigorous solution of the ***mixed*** conductive-radiative heat transfer in absorbing and transmitting medium is very complicated and cumbersome (see e.g. [3], pp.446-451).

Exact solution of the mixed conductive-radiative transfer problem can only be obtained numerically and is not necessary to describe heat flux adequately in insulation materials. Generally, a simplified model can be used [5]. The model fits the data well if the optical parameters are properly chosen. Two-flow model is most common, either with a single absorption parameter, or with separate absorption and scattering parameters. A diffusion model is also frequently used, usually with absorption only, or with isotropic scattering.

All of these models use absorption and scattering parameters, which are averaged over incident and scattering angles according to different prescriptions. For thermal transport problems the model equations can be averaged over wavelength if thermal weight functions are used to integrate the wavelength-dependent cross-sections [5]. Some numerical models for coupled conduction and radiation steady-state heat transfer in insulation materials were developed and comparison with experimental data was made in [6], [7] and [8].

Radiative heat flux (energy per unit time and area) q_{rad} in a medium, which is partially opaque, is often expressed in the form [6]

$$q_{rad} = (-grad\ T)\ 4\sigma n^2\ T^3 l$$

where σ is Stefan-Boltzmann constant, T is absolute temperature, n is average index of refraction, and l is the mean free path of photons.

The mean free path of photons is limited by absorption and scattering inside the medium, as well as by the finite dimensions of the medium. However, scattering and absorption cross-sections usually vary with the photon frequency f, so one must use in place of l a spectral integral, i.e.:

$$\int_0^\infty z^4 e^z\, l(z)/(e^z-1)^2 dz \Big/ \int_0^\infty z^4 e^z/(e^z-1)^2 dz$$

where $z=hf/kT$ is the reduced dimensionless frequency, h is Planck constant, (h= 6.626x10^{-34} J sec), k is Boltzmann constant (k=1.381x10^{-23} J/K). Integral in denominator is equal to ≈26 [6].

For a slab of thickness Δx with a temperature gradient normal to the slab, comparing l with the standard expression for the radiative heat transport between two parallel and perfectly emitting surfaces, one finds $l_B=4\Delta x/3$.

Absorption, scattering and the external dimensions act additively to limit l:

$$l(f)=1 / [1/l_a(f) + 1/l_s(f) + 1/l_B]$$

where l_a is the mean free path due to absorption, l_s is scattering mean free path, and l_B is boundary mean free path [6].

In general, almost all thermal conductivity test methods actually measure the thermal resistance or conductance of a specimen for a particular set of test conditions. In fact, these can be sufficient to characterize insulating material well enough for their selection for the proper end use: the "true" thermal conductivity is usually not necessary. If a comparison is desired between different specimens, then it is necessary that measurements be made for the same thickness and temperature difference and that these be typical of the applications considered for the materials [9].

References

1. Hollingsworth, M., Jr., "Experimental Determination of the Thickness Effect in Glass Fiber Building Insulation" *Thermal Insulation Performance, ASTM STP 718,* D.L.McElroy and R.P.Tye, Eds., American Society for Testing and Materials, 1980, pp.255-271.
2. Pelanne, C.M., "Discussion on Experiments to Separate the "Effect of Thickness" from Systematic Equipment Errors in Thermal Transmission Measurements," *Thermal Insulation Performance, ASTM STP 718,* D.L.McElroy and R.P.Tye, Eds., American Society for Testing and Materials, 1980, pp.322-334.
3. Holman, J.P., "Heat Transfer" 8th ed., McGraw-Hill, Inc., New York etc., 1997, p.430.
4. Flynn, D.R. and Gorthala, R., "Radiation Scattering versus Radiation Absorption – Effects on Performance of Thermal Insulation under Non-Steady-State Conditions," *Insulation Materials: Testing and Applications: Third volume, ASTM STP 1320*, R.S.Graves and R.R.Zarr, Eds., American Society for Testing and Materials, 1997, pp.366-380.
5. McKay, N.L., Timusk, T. and Farnaworth, B., "Optical Properties and Radiative Heat Transport in Polyester Fiber Insulation," *Thermal Conductivity 18*, New York – London, 1983, pp.393-402.
6. Klemens, P.G. and Kim, N., "Radiative Heat Transfer in Inhomogeneous Media and Insulations," *Thermal Conductivity 19*, Plenum Press, New York – London, 1985, pp.453-458.
7. Rish III, J.W., Roux, J.A. and Fairey III, P.W., "The Resistance of Fibrous Insulations Undergoing Coupled Transient Conduction and Radiation Heat Transfer," *Thermal Conductivity 19*, Plenum Press, New York – London, 1985, pp.497-510.
8. Yajnik, S. and Roux, J.A., "Apparent Thermal Conductivity of High Density and Low Density Fiberglass Insulations," *Thermal Conductivity 20*, Plenum Press, New York – London, 1987, pp.25-39.
9. ASTM Subcommittee C16.30, "What Property Do We Measure?" *Heat Transmission Measurements in Thermal Insulations, ASTM STP 544*, American Society for Testing and Materials, 1974, pp.5-12.